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Measurements of Thermal Diffusivity for Thin Slabs by a Converging Thermal Wave Technique[1]

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The measurement of thermal diffusivity for thin slabs by a converging thermal wave technique has been studied. Temperature variation at the center of the heat source ring that is produced by a pulsed high-power laser is detected by an infrared detector. A computer program based on the finite difference method is developed to analyze the thermal diffusivity of the slabs. Materials of both high thermal diffusivity (CVD diamond wafer) and low thermal diffusivity (stainless-steel foil) have been used for the measurements. The measurements have been performed by varying the size and the thickness of specimen. The converging thermal wave technique has proved to be a good method to measure the thermal diffusivity of a CVD diamond without breaking the wafer into small specimens. The technique can be applied for a small slab if the diameter of the slab is two times larger than that of the heat source ring. The sensitivity of thickness in measuring the thermal diffusivity is low for ordinary CVD diamond. The use of the converging thermal wave technique for nonhomogeneous, nonuniform, and anisotropic materials has been accomplished by applying the finite difference method.

KEY WORDS: converging thermal wave technique; CVD diamond; finite difference method; thermal diffusivity.

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1. INTRODUCTION

As industry is developing, the use of thin slabs for thermal management applications is growing. For example, thin large-area chemical vapor deposition (CVD) diamond wafers are used as a heat spreader and a heat management electrical insulator for high-power electronic packages. Above room temperature, diamond has the highest thermal conductivity of known materials. The thermal diffusivity is one of the most important physical properties for thermal applications. Many techniques have been suggested to measure the thermal diffusivities of thin slabs, such as the three-omega technique [1], the photothermal technique [2], the laser-flash technique [3], and the dc-heated bar technique [4]. However, these techniques are time-consuming in preparation of the specimen and in measurements. Some techniques need to cut the wafer to a smaller specimen for the measurement. The measured results of the others vary sensitively with a small change in the thickness of specimen.

A converging thermal wave technique was suggested to overcome the disadvantages of the other methods [5, 6]. The main advantage of the converging thermal wave technique is that the thermal diffusivity can be measured without breaking a thin slab into small specimens. Moreover, this technique is a noncontact measuring method, which uses a laser beam as the heat source and uses an IR detector as the temperature measurement tool. When analyzing the thermal diffusivity from the measured temperature variation, however, previous studies assumed the specimen to be a very thin and infinitely large plate. This assumption may cause an uncertainty for the analysis of relatively thick slabs such as a CVD diamond wafer.

In this study, the temperature variations of a CVD diamond wafer and various metal foils are measured by a converging thermal wave technique. A computer program and an analytic method have been developed to analyze the thermal diffusivity from the measured data without assuming that the plate is very thin and infinitely large. The effects of the thickness and size of the specimen on applying the converging thermal wave technique have been studied. The possibility of applying this technique not only to a homogeneous, uniform, and isotropic material such as a diamond wafer but also to nonhomogeneous, nonuniform, and anisotropic materials such as coated materials has been proposed.

2. EXPERIMENTAL APPARATUS AND PROCEDURES

A converging thermal wave technique is adopted to measure the in-plane thermal diffusivity of thin slabs without breaking. A pulsed Nd:Yag laser beam with an energy of 0.9 J/pulse, a wavelength of 1.06 μm, a diameter of 10 mm, and a pulse width of 0.35 ms is passed through a convex lens and an axicon to produce an annular ring of heat source. The diameter of the annular heating ring is 9.0 to 10.0 mm on the slab surface, and the width of the annular ring is a few tens of micrometers. Temperature excursions at the center of the heating ring are measured with an IR detector at the rear side of the thin slabs. Dried graphite fluid is sprayed on both surfaces to obtain a uniform emissivity. The measurements are performed on a CVD diamond wafer with a thickness of 1 mm and on various metal foils of thickness 50 μm. Detailed experimental procedures are given by Chae et al. [7].

3. MODELING AND ANALYTIC METHOD

3.1. Modeling

A diamond wafer is installed vertically in air at room temperature. Considering that the thermal conductivity of air is 10^{-5} that of diamond and the measurements are performed over the very short time period of 10 ms, the heat loss by air conduction and convection can be neglected. Even the energy density of the heat source ring is high at the moment of the laser beam's irradiation; the density decreases rapidly as the heat diffuses in the wafer. Because the temperature increase in the wafer due to the laser pulse is small, the heat loss from each side of the wafer by thermal radiation can be neglected. The temperature measurements are affected by the region near the heat source ring, for which the diameter is 9.0 to 10.0 mm. Since the diameter of the wafer is 100 mm and the thickness is 1 mm, the wafer can be considered to be a thin, infinitely large object. Because the properties of the wafer are relatively uniform through the entire wafer, the properties around the heat source ring can be assumed to be constant. Therefore, the heat is assumed to flow in the radial and vertical directions and not to flow in the azimuthal direction.

3.2. Analytic Method

Under the assumptions of Section 3.1, the heat conduction equation can be simplified to the following two-dimensional, unsteady-state equation.

$$\frac{1}{\alpha}\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2} \qquad (1)$$

where α is the thermal diffusivity, T is the temperature of the wafer, t is the time, r is the radius from the center of the heat source ring, and z is the height from the surface where the temperature is measured by an infrared detector (the opposite side of the surface where the laser beam is irradiated). To make this equation dimensionless, the following dimensionless parameters were chosen.

$$r^* = \frac{r}{R}, \qquad z^* = \frac{z}{e}, \qquad t^* = \frac{\alpha t}{R^2} \tag{2}$$

where R is the radius of the heat source ring and e is the thickness of the wafer. Let $A = e/R$; then a dimensionless governing equation is obtained as

$$\frac{\partial T}{\partial t^*} = \frac{\partial^2 T}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial T}{\partial r^*} + \frac{1}{A^2}\frac{\partial^2 T}{\partial z^{*2}} \tag{3}$$

The boundary conditions and the initial conditions are as follows.

$$\frac{dT}{dz^*} = 0 \qquad \text{at} \quad z^* = 0 \qquad \text{and} \qquad z^* = 1 \qquad \text{for} \quad t^* > 0 \tag{4}$$

$$T \text{ is finite} \qquad \text{at} \quad r^* = 0 \qquad \text{for} \quad t^* > 0 \tag{5}$$

$$\begin{aligned} T(r^*, z^*, 0) &= T_0 \qquad \text{at} \quad r^* = 1 \qquad \text{and} \qquad z^* = 1 \\ &= 0 \qquad \text{at} \quad r^* \neq 1 \qquad \text{and} \qquad z^* \neq 1 \end{aligned} \tag{6}$$

where T_0 is the initial temperature at the heat source ring.

Solving Eq. (3) with the boundary and initial conditions of Eqs. (4)–(6) by applying the separation of variables, the temperature distribution of the wafer with time is obtained as follows [8].

$$\begin{aligned} T(r^*, z^*, t^*) = T_0 \times \frac{1}{2t^*} \exp\left(-\frac{1 + r^*}{4t^*}\right) \times I_0\left(\frac{r^*}{2t^*}\right) \\ \times \left\{1 + \sum_{n=1}^{\infty} 2 \cos \eta_n \cos \eta_n z^* \exp\left(-\frac{\eta_n^2}{A^2} t^*\right)\right\} \end{aligned} \tag{7}$$

The temperature at the origin ($r^* = 0, z^* = 0$), where the temperature is measured by an infrared detector, can be simplified as

$$T(0, 0, t^*) = T_0 \times \frac{1}{2t^*} \exp\left(-\frac{1}{4t^*}\right) \times \left\{1 + \sum_{n=1}^{\infty} 2 \cos n\pi \exp\left(-\frac{n^2\pi^2}{A^2} t^*\right)\right\} \tag{8}$$

3.3. Finite Difference Method

Equation (1) is a two-dimensional parabolic equation. Many numerical methods have been proposed to solve two-dimensional parabolic equation problems, but among them the alternating direction implicit (ADI) method is the most frequently used because it is unconditionally stable [9]. The ADI method is applied to Eq. (1). A time step is divided by two, and for the first half-time step, the following finite difference equation is applied in the horizontal direction.

$$\frac{T_{i,j}^{n+1/2}-T_{i,j}^{n}}{\alpha\,\Delta t/2}=\frac{1}{r_i}\,\frac{T_{i+1,j}^{n+1/2}-T_{i-1,j}^{n+1/2}}{2\Delta r}+\frac{T_{i-1,j}^{n+1/2}-2T_{i,j}^{n+1/2}+T_{i+1,j}^{n+1/2}}{\Delta r^2}+\frac{T_{i,j-1}^{n}-2T_{i,j}^{n}+T_{i,j+1}^{n}}{\Delta z^2} \tag{9}$$

Then, for the second half-time step, the following equation is applied in the vertical direction.

$$\frac{T_{i,j}^{n+1}-T_{i,j}^{n+1/2}}{\alpha\,\Delta t/2}=\frac{1}{r_i}\,\frac{T_{i+1,j}^{n+1/2}-T_{i-1,j}^{n+1/2}}{2\Delta r}+\frac{T_{i-1,j}^{n+1/2}-2T_{i,j}^{n+1/2}+T_{i+1,j}^{n+1/2}}{\Delta r^2}+\frac{T_{i,j-1}^{n+1}-2T_{i,j}^{n+1}+T_{i,j+1}^{n+1}}{\Delta z^2} \tag{10}$$

At the center of the heat source ring ($r=0$), the first terms on the right-hand sides of Eqs. (9) and (10) are 1/0 and become singular. Therefore, Eqs. (9) and (10) cannot be applied at the meshes of $r=0$. To circumvent the singularity problem, the following is applied to Eq. (1).

$$\left(\frac{1}{r}\frac{\partial T}{\partial r}\right)_{r=0}=\frac{(\partial/\partial r)(\partial T/\partial r)}{(\partial/\partial r)\,r}=\left(\frac{\partial^2 T}{\partial r^2}\right)_{r=0} \tag{11}$$

Then Eq. (1) becomes

$$\frac{1}{\alpha}\frac{\partial T}{\partial t}=2\,\frac{\partial^2 T}{\partial r^2}+\frac{\partial^2 T}{\partial z^2} \tag{12}$$

Equation (12) is discretized with the same method as Eqs. (9) and (10) and is applied at the origin.

The area where the temperature varies due to the heat irradiated from the laser beam expands continuously with time. However, the computational area cannot be infinitely large. The computational area is determined up to the area where heat is spread during the considered time period. The end of the computational area is assumed to be insulated. For a real calculation, the radius of the computational area is about three times larger than that of the heat source ring.

4. RESULTS AND DISCUSSION

4.1. CVD Diamond Wafer

Heat expansion in the CVD diamond wafer just after the laser beam is irradiated on the heat source ring is calculated by the finite difference method, and the results are shown in Fig. 1. Before the laser beam is irradiated, the temperature of the whole wafer is assumed to be 0. At $t = 0$, the laser is irradiated and the temperature of the heat source ring ($r = 4.4775$ mm, $z = 1$ mm) is assumed to become 1. The thermal diffusivity of the wafer is

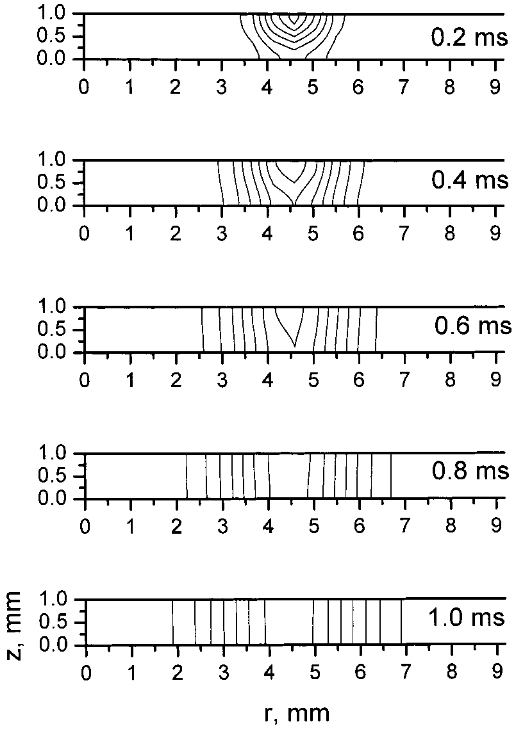

Fig. 1. Temperature profile of the wafer calculated by the finite difference method. The laser beam is irradiated at the heat source ring ($r = 4.4775$ mm, $z = 1$ mm) at $t = 0$.

assumed to be 690 $\mathrm{mm}^2 \cdot \mathrm{s}^{-1}$. The absolute value of the temperature varies with the size of mesh, but the characteristics of the temperature distribution are independent of the mesh size. The temperature distribution in the circular area of 17.91-mm radius was calculated. Figure 1 shows temperature distributions at 0.2-ms increments from 0.2 to 1.0 ms. At 0.2 ms after the laser beam is irradiated, isothermal lines are formed in a concentric circular shape, the center of which is the heat source ring. It is known that the heat is spread uniformly in the radial and thickness directions. The heat spreads in both the inner and the outer directions. The diffusion velocity in the inner direction is higher than that in the outer direction. This is because the cross section decreases as the heat spreads in the inner direction, while it increases as the heat spreads in the outer direction. At 0.8 ms, the isothermal lines are shown as almost-vertical lines. After this time, most of the heat is spread in the radial direction, and the temperature is almost uniform in the thickness direction.

The temperature distributions of the top surface of the wafer are calculated by the finite difference method, and the results are shown in Fig. 2. Curves are shown at 1-ms increments from 1 to 10 ms. The x coordinate is the distance from the center of the heat source ring, and the y coordinate represents the temperature. Because the temperature is known to be

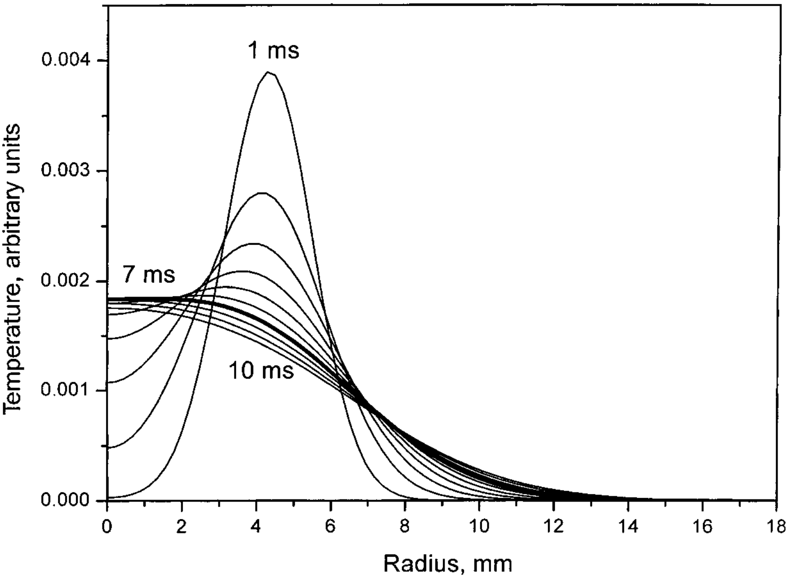

Fig. 2. Temperature distribution of the top of the wafer calculated by the finite difference method.

uniform in the vertical direction after 0.8 ms, the temperature distributions in Fig. 2 represent that of the same radius. The curve that shows the highest value at the heat source ring is the temperature distribution at 1 ms. The peak value of the temperature decreases rapidly, from a value of 1 at $t=0$ to a value of 0.004 at $t=1$ ms. The point of the highest temperature moves one mesh in the inner direction, from 4.4775 to 4.2536 mm. This means that the heat diffusion rate in the inner direction is faster than that in the outer direction. Until this time, heat does not reach the center, and the temperature of the center does not vary. For larger times, the point where the highest temperature occurs moves in the inner direction, and the peak value decreases gradually and the curve becomes blunt. The heat diffused to the outside expands continuously. However, the heat diffused in the inner direction accumulates at the center, and the temperature begins to increase rapidly. The thick curve in Fig. 2 represents the temperature distribution at 7 ms. The temperature at the center reaches the highest value at this time, and the temperature at the center is the highest over the whole wafer. (The exact time of this event is 7.28 ms.) As the time increases, the heat diffused to the inner side of the heat source ring spreads in the outer direction, and the temperature at the center decreases gradually.

The time variations of temperature at the center are calculated by the finite difference method and by the analytic method. Figure 3 shows the temperature distributions of the measured data. The x coordinate represents the time after the laser beam is irradiated, and the y coordinate represents the temperature of the center, which is normalized by the peak value at the center. The thick solid curve is the temperature distribution that is calculated by the finite difference method for the case of a wafer thickness of 1 mm, a heat source ring of radius 4.4775 mm, and a thermal diffusivity of $690\,\mathrm{mm}^2\cdot\mathrm{s}^{-1}$. Calculations were performed for the period until the heat reached to the outer meshes. There is almost no temperature variation at the center of the heat source ring until the heat irradiated from the laser reaches the center. However, the temperature increases very rapidly after the heat reaches to the origin. The temperature reaches its maximum value at 7.28 ms, and then the temperature at the center decreases slowly as the heat spreads to the surroundings. The dashed line is the temperature variation calculated by the analytic methods of Eq. (8) for the same parameters as with the finite difference method. The temperature variations obtained by these two methods agree quite well. The times for the peak temperatures at the center are 7.28 and 7.16 ms, respectively, and show only 1.6 % difference. The experimental data shown in Fig. 3 are the temperature variation at the center of the heat source ring of radius 4.4775 mm detected by the IR detector for the wafer with a thickness of 1 mm. Because the measured data agree well with the predicted data by

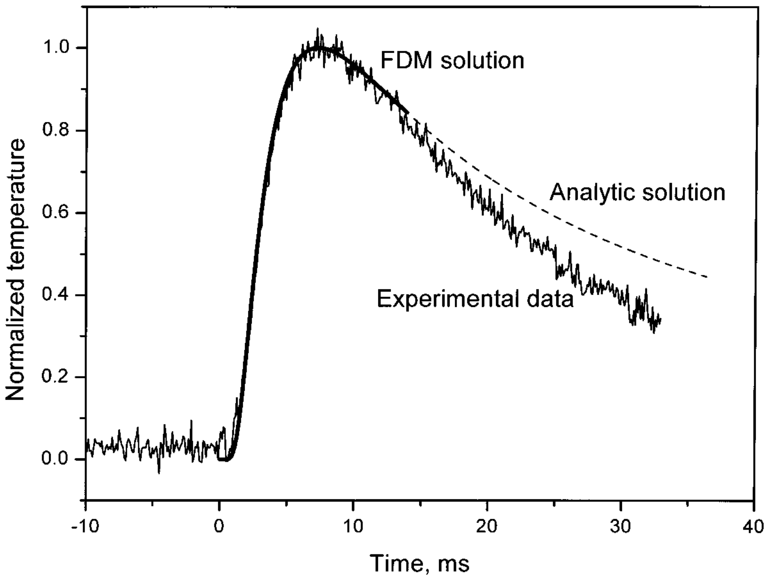

Fig. 3. Temperature variations at the center of the wafer obtained by the finite difference method, the analytic method, and measurements.

the finite difference method or by the analytic method, the heat diffusivity of the wafer is determined to be 690 $\mathrm{mm}^2 \cdot \mathrm{s}^{-1}$.

Comparing temperature–time measurement results to the values predicted by the finite difference method by trial and error, the best-matched thermal diffusivity is determined for the thermal diffusivity of the diamond wafer. The thermal diffusivities of a whole CVD diamond wafer are measured by this converging thermal wave technique. The detailed results are given by Chae et al. [7].

The sensitivity to specimen thickness for the analysis of thermal diffusivity was studied. For a typical value of the thermal diffusivity of a CVD diamond wafer, 600 $\mathrm{mm}^2 \cdot \mathrm{s}^{-1}$, the time at which the peak value is reached was calculated by the finite difference method while varying the thickness of the wafer. The time at which the peak value occurs appears to increase if the wafer thickness is larger than 2.5 mm. Thus, if the thickness is less than 2.5 mm and the thermal diffusivity is higher than 600 $\mathrm{mm}^2 \cdot \mathrm{s}^{-1}$, the thickness has almost no effect on the thermal diffusivity. Because the thickness of common CVD diamond wafers is about 1 mm, the converging thermal wave technique gives reliable results even if the thickness of the wafer varies a little.

4.2. Metal Foils

The thermal diffusivities of copper, aluminum, nickel, and stainless-steel foils are measured by the converging thermal wave technique. Figure 4 compares the measured thermal diffusivities with literature values. The thicknesses of the foils are 50 μm, and the purities are above 99.99%. The metal foils of high thermal diffusivity, i.e., copper, aluminum, and nickel, show a good agreement with literature values, with differences of 3 to 8%. However, stainless steel, a low-thermal diffusivity metal, shows the relatively large difference of 15%. One of the reasons is believed to be that because the measurement of the low-thermal diffusivity metal takes longer than that of the high-thermal diffusivity metals, the assumption of no heat loss from the sides of the foil may cause a larger error. Because the measurement of a CVD diamond wafer is completed within much less time than that of the metal foils, the error due to the heat loss from the sides of the diamond wafer is much smaller. The time at which the peak value is reached for stainless steel is about 1450 ms, but that for CVD diamond is 7 ms.

The thermal diffusivities of various specimen sizes are measured and calculated by the finite difference method. Figure 5 shows the time at which

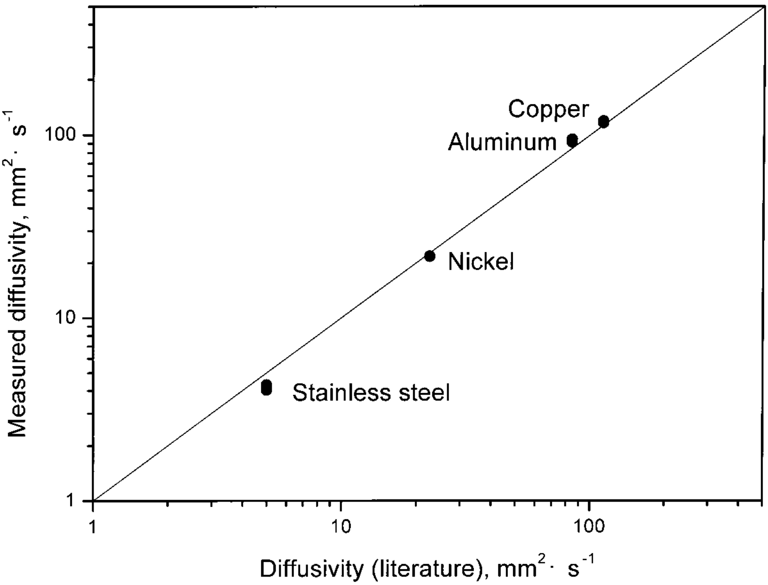

Fig. 4. Thermal diffusivities of various metal foils measured by the converging thermal wave technique, along with literature values. Foil thickness, 50 μm.

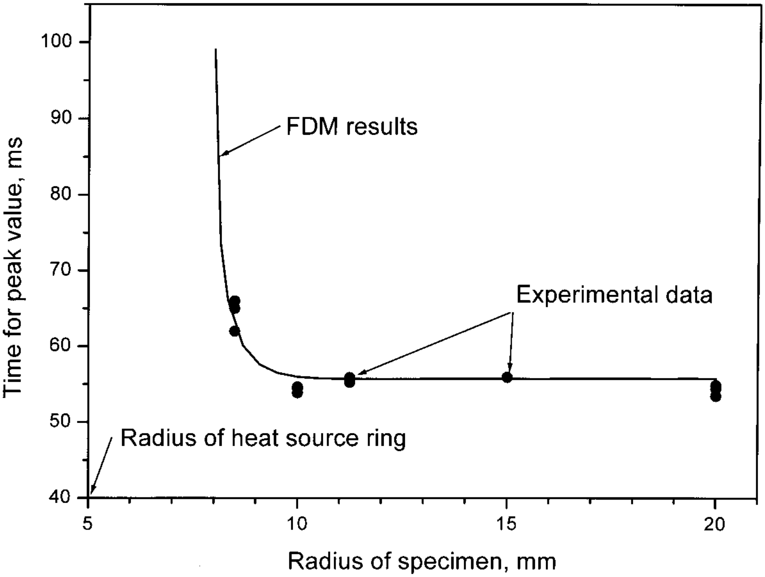

Fig. 5. Time for the peak value of various-sized copper foils measured by the converging thermal wave technique and that calculated by the finite difference method.

the peak value of copper foils is reached, for which the radius is reduced from 20 to 8.5 mm. The radius of the heat source ring is 5.0 mm. Until the radius of the specimen is reduced by up to twice that of the heat source ring, the peak time does not vary significantly. However, when the radius of the specimen is reduced by 1.7 times that of the heat source ring, the peak time increases. The peak times predicted by the finite difference method show a good agreement with the measured values. The peak time increases very rapidly when the radius is reduced to less than a factor of 1.9 that of the heat source ring. The converging thermal wave technique is known to apply for thin and large area slabs. From this result it is known that if the diameter of the specimen is, at least, two times larger than that of the heat source ring, the converging thermal wave technique gives a reasonable value of the thermal diffusivity.

The time at which the peak value is reached increases rapidly when the heat source ring approaches the edge of the specimen. The increase in the time for the peak value means a decrease in the thermal diffusivity. This result can be used to detect a possible crack in a CVD diamond wafer, which is hard to observe with the naked eye. If a region in the diamond wafer shows a sudden decrease in thermal diffusivity, a crack may exist

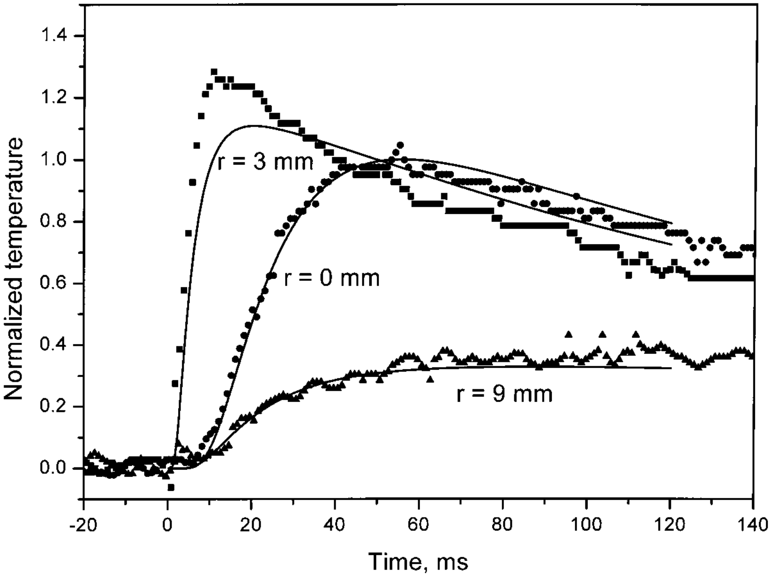

Fig. 6. Temperature variation at different points of a copper foil.

near the heat source ring. By drawing a thermal diffusivity contour map, the position of a crack can be detected.

The time variations of temperature at off-center points of the heat source ring are measured and calculated by the finite difference method. Figure 6 shows the temperature variations of some of the points, the center ($r = 0$ mm), a point inside the heat source ring ($r = 3$ mm), and a point outside the heat source ring ($r = 9$ mm). The x coordinate represents time and the y coordinate represents temperature normalized by the peak temperature at the center. Figure 6 shows that the measured and calculated values agree well.

4.3. Comparison of the Analyses by the Analytic and Finite Difference Methods

The analytic method is simpler, but it can be applied only to plates of infinitely large area. The temperature at the center can be obtained relatively easily, but the temperature distribution at the other points is very difficult to determine.

On the other hand, the finite difference method can be applied to specimens of any size. The method can be used to calculate the temperature of the specimen at any point relatively easily. Moreover, by changing the material properties of each node, the finite difference method can be applied for analysis of the thermal diffusivity of nonhomogeneous, nonuniform, and anisotropic materials such as composite materials. It also can be applied to a specimen of two-layered materials, such as a metal plate coated with a diamond-like carbon.

5. CONCLUSION

A converging thermal wave technique has been developed to measure the thermal diffusivity of thin slabs. The technique has proved to be a good method to measure the thermal diffusivity of a CVD diamond without breaking the wafer into small specimens.

A finite difference method model was developed for analysis of the measurement results. By applying the finite difference method, it was shown that the converging thermal wave technique can be applied for a thin small slab as long as the diameter of the slab is two times larger than that of the heat source ring. The sensitivity of the thermal diffusivity to the thickness has been shown to be very small for ordinary CVD diamond wafers. By applying the finite difference method, the converging thermal wave technique can be used not only for homogeneous uniform isotropic materials but also for nonhomogeneous nonuniform anisotropic materials.

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